

Joint U.S.-Russian Conference on Advances in Materials Science

Accurate Prediction of Dynamic Fracture with Peridynamics

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Prague, Czech Republic August 30 – September 3, 2009





Fracture Mechanics Theory and Dynamic Fracture

- Onset of crack growth can be accurately predicted
- Crack growth speed and direction cannot!

Wanted: A successful method for simulating <u>Dynamic Fracture</u>

- Such a method must be able to reproduce the characteristic phenomena of Dynamic Fracture.
- All current simulation methods severely fail this test of efficacy
 - The governing PDEs break down at cracks.



Characteristic Phenomena of Dynamic Fracture:

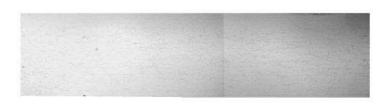
- 1. The Mirror, Mist, Hackle sequence of textures on the fracture surface;
- 2. A steady, limiting crack speed;
- 3. The transition from stable to unstable crack growth;
- 4. Crack branching;
- 5. Fragment size distribution;
- 6. The specific angle of cracking following impact of a notched plate (the Kaltoff-Winkler experiment);
- 7. The multiple, unstable cracking modes of fiber-reinforced composites;
- 8. Membrane bursting;
- 9. Unstable peeling and tearing of thin sheets.



1, 2. Dynamic fracture in PMMA:

J. Fineberg & M. Marder, Phys Rpts 313 (1999) 1-108

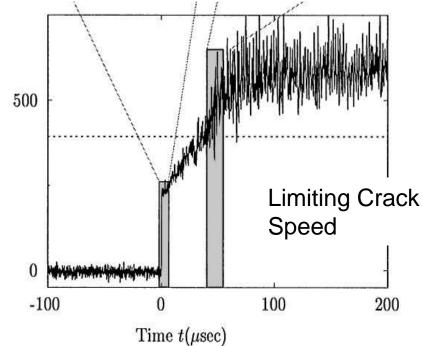
Velocity $v \times (m/sec)$







Mirror-mist-hackle transition

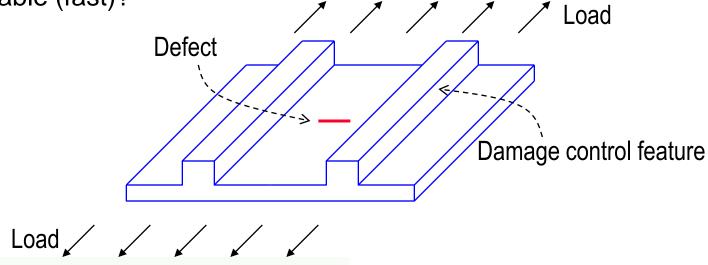




3. Transition from Stable to Unstable Crack Growth

At what load and what length does crack growth change from stable (slow)

to unstable (fast)?



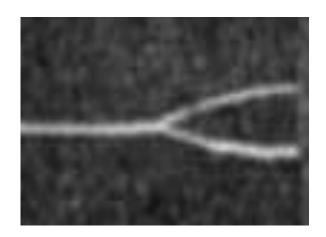


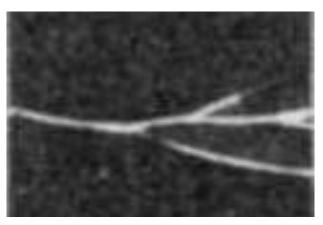
Crack just before transition to Unstable (photo courtesy Boeing)

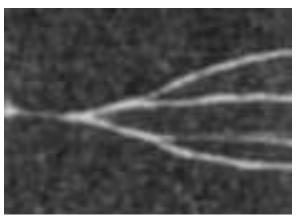


4. Crack Branching

At high levels of stress intensity factor (e.g., at a round notch) a crack propagates at higher stress levels and sequential crack branching can occur.



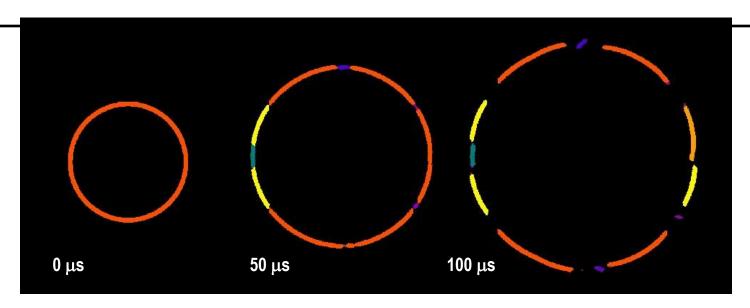




Crack branching in notched specimens of soda-lime glass, pulled vertically (specimens 3" x 1" x 0.05"; load stress increasing left to right)

S F. Bowden, J. Brunton, J. Field, and A. Heyes, *Controlled fracture of brittle solids and the interuption of electrical current*, Nature (1967) 216, 42, pp.38-42.

5. Fragmentation: Ductile Aluminum Ring*



1100-0 Al ring has initial radial velocity of 200m/s.

- 32mm diameter, 1mm x 1mm cross-section.
- Rapid acceleration by electromagnetic pulse.
- Recovered 11-13 fragments per shot over range of initial velocities 182 – 220 m/s.

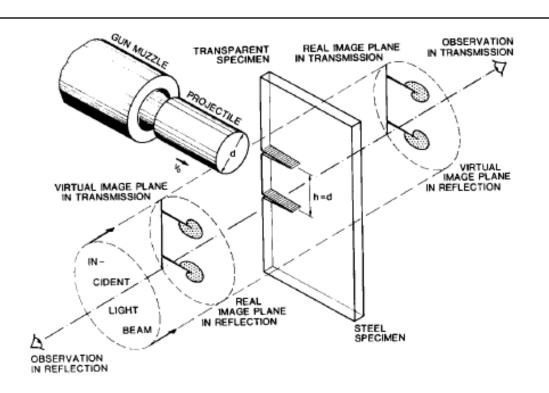
*D. E. Grady & D. A. Benson, Experimental Mechanics 23 (1983) 393-400

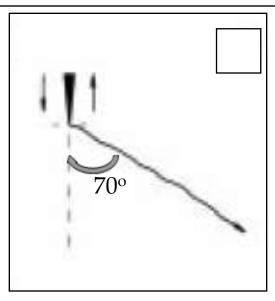


6. **Dynamic fracture in a hard steel plate:**Kalthoff-Winkler Experiment

Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)

• Mode-II loading at notch tips results in mode-I cracks at 70° angle.





Experimental Results



7. Splitting and fracture mode changes in Fiber-reinforced Composites

The distribution of fiber directions between plies strongly influences the way cracks grow.



Typical crack growth in a notched laminate (photo courtesy Boeing)



8. Dynamic fracture in membranes



Bursting of pressurized membrane by a bullet: High speed photography of H. Edgerton (MIT collection)

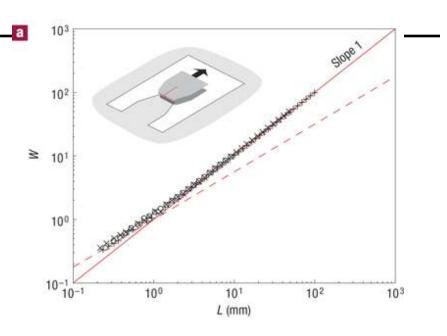
(http://mit.edu/6.933/www/Fall2000/edgerton/edgerton.ppt). Time increases right to left.



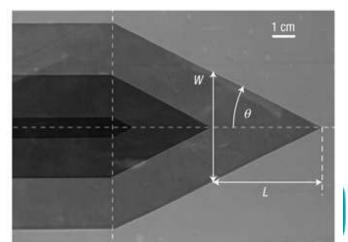
9a. Peeling and Tearing

b

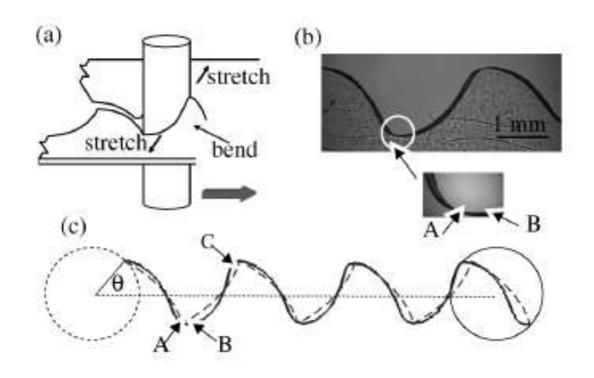
Pealing of adhesive film from rigid substrate exhibits characteristic tearing behavior.



E. Hamm et al, Nature Materials 7 (2008) 386-390



9b. Instability in the slow tearing of an elastic membrane



Ghatak & Mahadevan, *Physical Review Letters* 91 (2003) 215507-1–2155-7-4



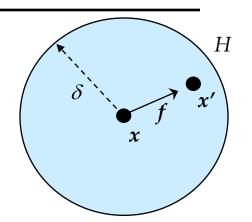
Peridynamics Governing Equation

• PDEs are replaced by the following integral equation:

$$\rho \ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \int_{H} \boldsymbol{f}(\boldsymbol{u}'-\boldsymbol{u},\boldsymbol{x}'-\boldsymbol{x})dV'-\boldsymbol{b}(\boldsymbol{x},t)(1)$$

• Eq.(1) replaces the classical PDE:

$$\rho \ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \nabla \bullet \boldsymbol{\sigma}(\boldsymbol{x},t) - \boldsymbol{b}(\boldsymbol{x},t) \tag{2}$$



- New approach is an extension of classical continuum mechanics:
 - PD, Eq.(1) is not derivable from Eq. (2);
 - PD converges to (2) in the limit $\delta \rightarrow 0$ (J.Elasticity 2008);
 - PD holds regardless of discontinuities;
 - PD entails force & displacement, not stress & strain;
 - PD includes interaction of material particles over a finite distance
- PD is intrinsically non-local,
- Eq. (1) is a "Continuum version of molecular dynamics"

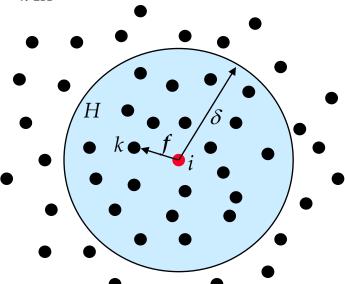


Numerical method and material model incorporate damage at the "bond" level

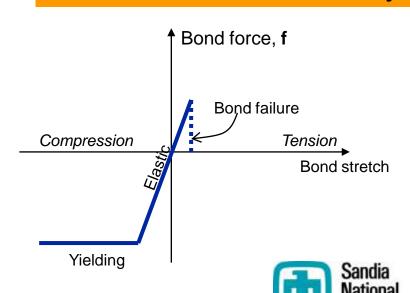
- Finite sum replaces integral: method is meshless and Lagrangian.
- Force parameters come from measurable elastic-plastic and fracture data.
- Simulate explicit time integration dynamics or static equilibrium; extension to implicit dynamics is underway.

$$\rho \ddot{\boldsymbol{u}}_{i}^{n} = \sum_{k \in H} f(\boldsymbol{u}_{k}^{n} - \boldsymbol{u}_{i}^{n}, \boldsymbol{x}_{k} - \boldsymbol{x}_{i}) \Delta V_{i} + \boldsymbol{b}(\boldsymbol{x}_{i}, t)$$

All <u>material-specific</u> behavior is contained in the force density, *f*.



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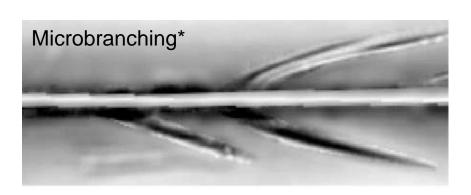


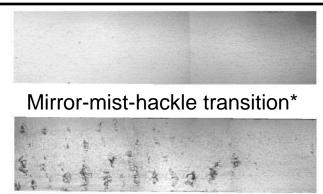
Fracture Phenomena *Emerge* from a Peridynamics Simulation

Peridynamics is a history-dependent theory in which crack initiation and growth, and all associated phenomena, emerge spontaneously, in an unquided fashion, simply from the choice of system geometry, ICs, BCs, and the constitutive model.

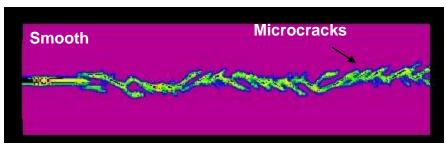


1. Dynamic fracture in PMMA: Damage features

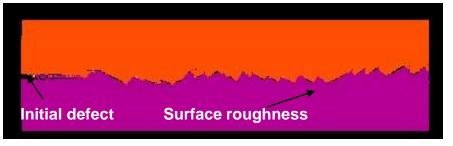








Peridynamics damage

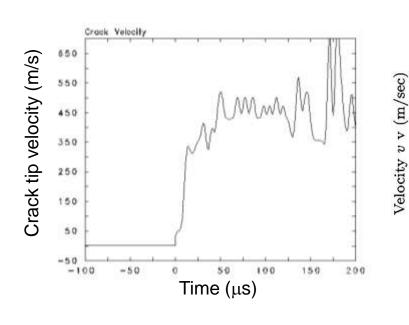


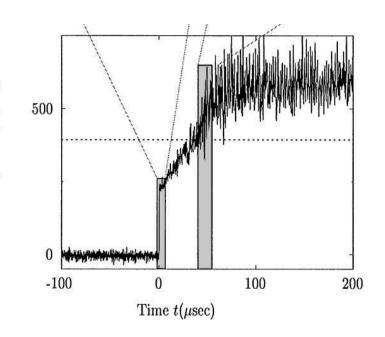
Peridynamics crack surfaces



2. Dynamic fracture in PMMA: Crack tip velocity

Crack velocity increases to a critical value, then oscillates.



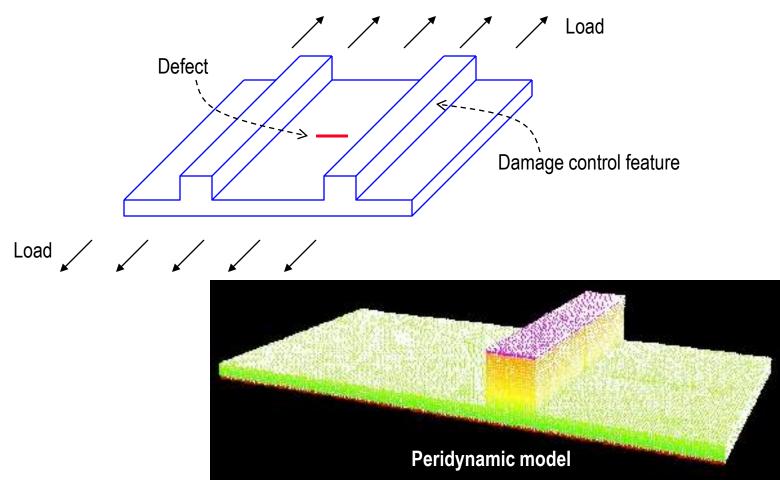


Peridynamics

Experiment*

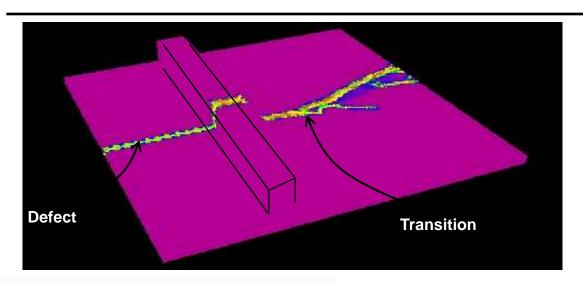


3. Peridynamics model of Ribbed Plate



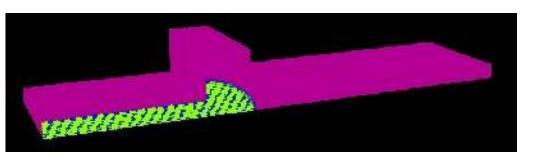


3. Simulation of damage control feature and crack instability



Crack trajectory after instability



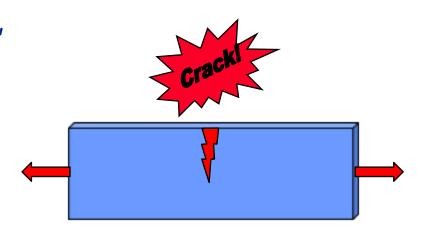


Crack just before transition to unstable (photo courtesy Boeing)



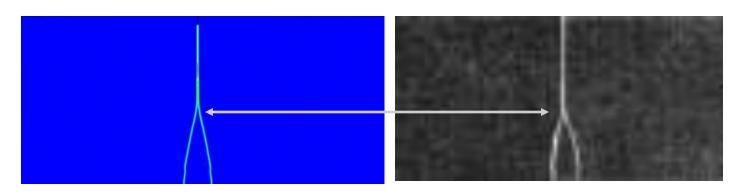
4. Dynamic Brittle Fracture in Glass

- Soda-lime glass plate
 - Dimensions: 3" x 1" x 0.05"
 - Density: 2.44 g/cm³
 - Elastic Modulus: 79.0 GPa
- Notch at top; apply tension
- Discretization (finest)
 - Mesh spacing: 35 microns
 - Approx. 82 million particles
 - Time: 50 microseconds (20k timesteps)
 - 6 hours on 65k cores using DAWN (LLNL): IBM BG/P System
 - 500 teraflops; 147,456 cores
- Simulation by M. Parks (SNL), F. Bobaru & Y. Ha (Nebraska)





4. Crack Branching in Brittle in Glass



Peridynamics Simulation

Physical Experiment*

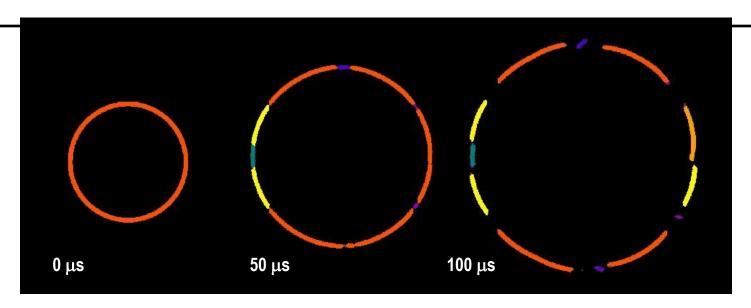
Crack branching *emerges* in the Peridynamics simulation of the experimental specimen and loading conditions:

- Branching is qualitatively identical to experiment;
- Onset of simulated branching is earlier than experiment.

*S F. Bowden, J. Brunton, J. Field, and A. Heyes, *Controlled fracture of brittle solids and the interuption of electrical current*, Nature (1967) 216, 42, pp.38-42.



5. Fragmentation of a Ductile Aluminum Ring*



Experiment *:

• Recovered 11-13 fragments per shot over range of initial velocities 182 – 220 m/s.

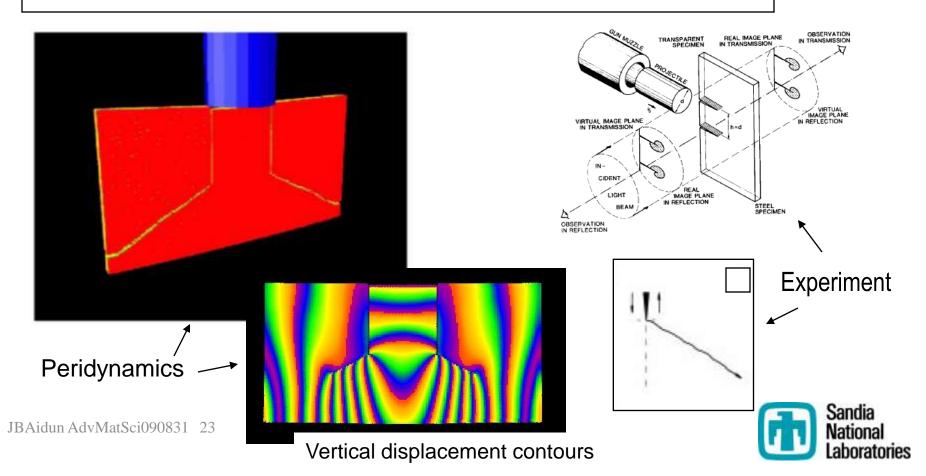
Peridynamics model produces 12 fragments.



6. Dynamic fracture in a hard steel plate

Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)

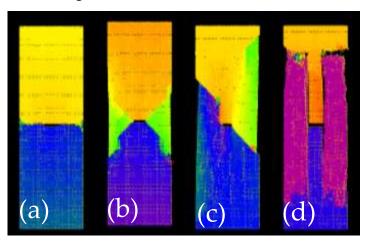
• 3D Peridynamics model reproduces the 70° crack angle.



7. Splitting and fracture mode changes in Fiber-reinforced Composites

• Distribution of fiber directions between plies strongly influences the way cracks grow.

Crack growth in a notched laminate bar



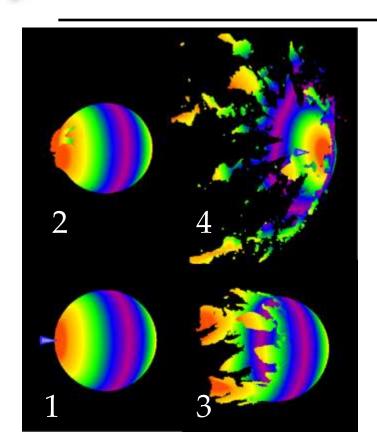
- a) Quasi-isotropic;
- b) Additional +/- 45°;
- c) Extra +/- 45°;
- d) Mostly 0° (along length)



Typical crack growth in a notched laminate (photo courtesy Boeing)



8. Dynamic fracture in membranes



Peridynamics model of a balloon penetrated by a fragment.

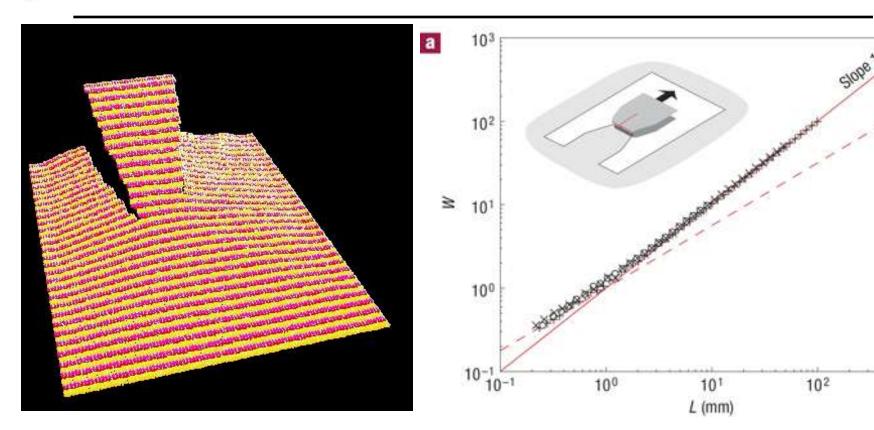
(Time sequence numbered.)



Bursting of a balloon by a bullet. Time increases *right to left*. High speed photo by H. Edgerton (MIT collection)



9a. Peeling and tearing



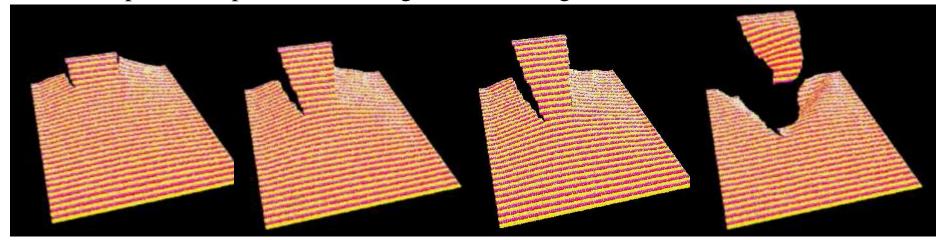
Peridynamics: Unsupported sheet, pinned along 3 edges (Computers and Structures, 2005)

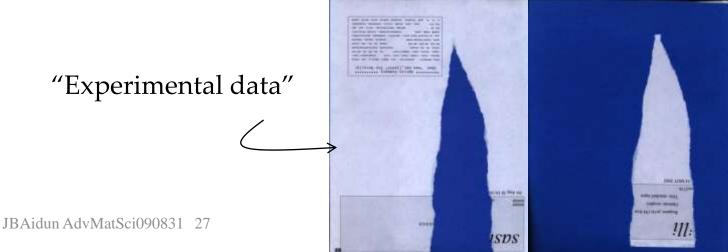
E. Hamm et al, Nature Materials 7 (2008) 386-390



9a. Interaction of 2 cracks: Peeling of a sheet

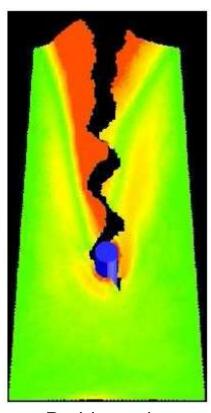
• Pull upward on part of a free edge – other 3 edges are fixed.



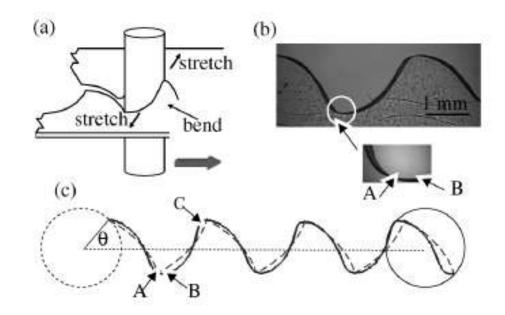




9b. Instability in the slow tearing of an elastic membrane



Peridynamics



Ghatak & Mahadevan, *Physical Review Letters* 91 (2003) 215507-1 – 2155-7-4

Silling & Bobaru, *Int'l Journal Non-Linear Mech.* 40 (2005) 395–409



Conclusion

Fracture phenomena spontaneously emerge in peridynamics simulations simply as a result of the choice of system geometry, ICs, BCs, and material model.

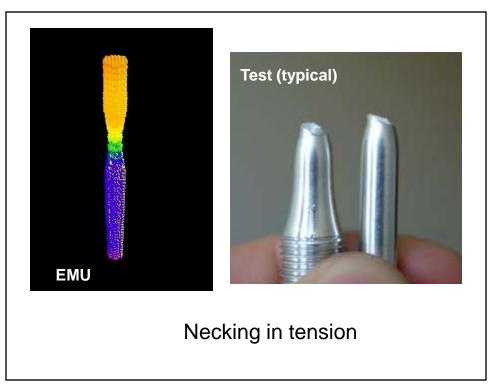
No supplemental kinetic relations are required; cracks grow in an unguided fashion.

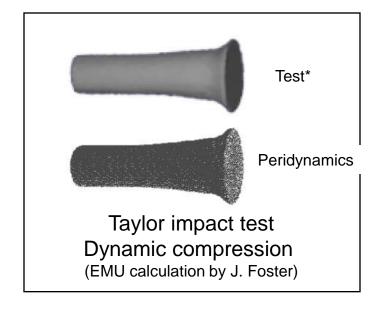
In this sense peridynamics is predicitive of dynamic fracture phenomena.



Peridynamic Can Use Diverse Constitutive Models

 Peridynamics can model large-strain, rate-dependent, strain-hardening plasticity.





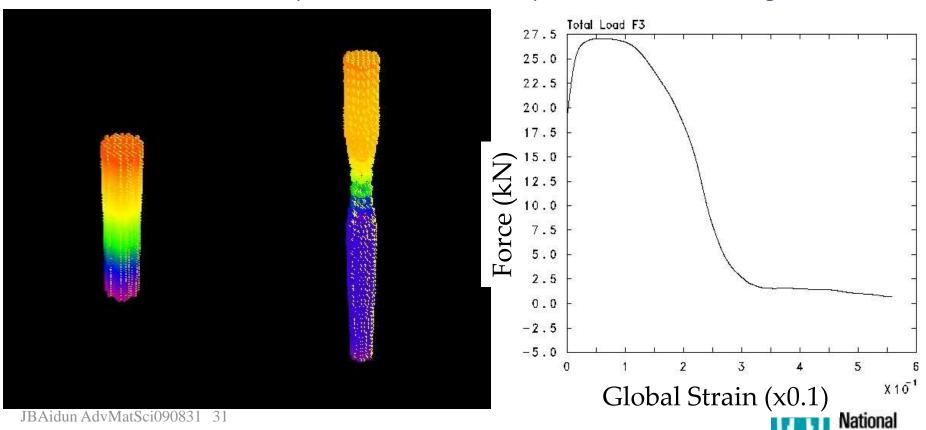
*C. E. Anderson Jr, A. E. Nicholls, I. S. Chocron, and R. A. Ryckman. Taylor anvil impact. In AIP Conference Proceedings, volume 845, page 1367. AIP, 2006



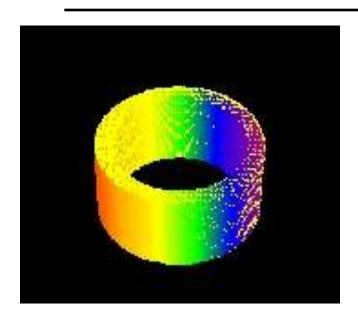
Necking in a 6061-Aluminum Bar

Simulation with state-based peridynamic implementation of large-deformation, strain-hardening, rate-dependent material model.

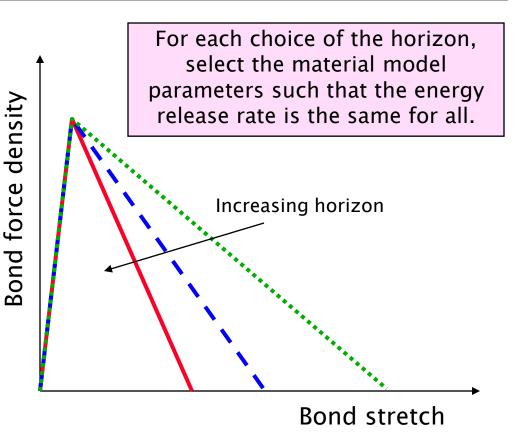
- Force-strain plot shows stable response with decreasing load.



Convergence in fragmentation: Expanding 3D annulus



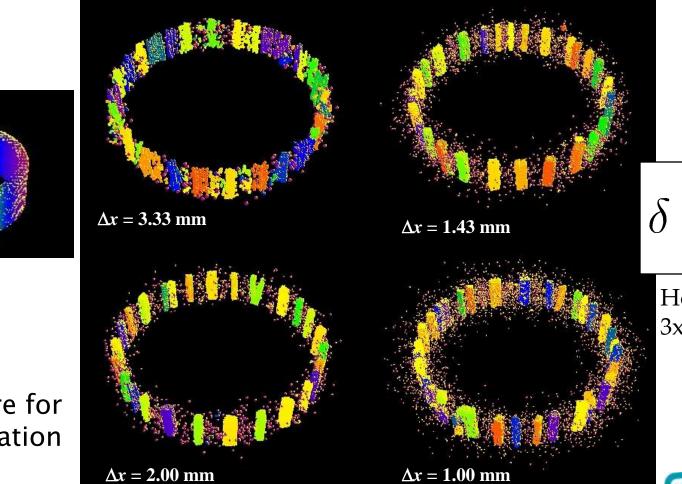
- •Outer radius = 100 mm
- •Thickness = 10 mm
- Radial velocity = 600 m/s



Grid contains 1% random perturbations to act as seed.



Convergence in fragmentation: Same problem with 4 different grid spacings



 $\delta = 3\Delta x$

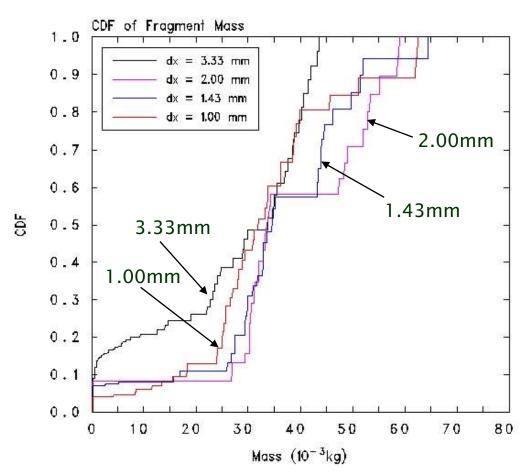
Horizon = 3x grid spacing

Colors are for visualization

Sandia National Laboratories

Convergence in fragmentation: Fragment mass distribution

Cumulative distribution function for 4 grid spacings



$\Delta x \text{ (mm)}$	Mean fragment mass (g)
3.33	27.1
2.00	37.8
1.43	35.9
1.00	1 33.5

Solution appears essentially converged

